Math Midway Class Activities

Museum of Mathematics

This guide is designed for museum educators who will host class trips to the Math Midway at their museum. A possible activity is outlined for each of the major Midway exhibits. The material is intended only to provide general models to inspire possible class workshops. Rather than follow these instructions line by line, educators are encouraged to draw from their own backgrounds to create an active student experience. Feel free to select from this material and adapt it to the particular circumstances of the students' abilities and the teachers' interests.

For more information about the Math Midway and the Museum of Mathematics, please see momath.org and mathmidway.org

Title: Ring of Fire Activity
Authors: Math Midway
Topic: Geometry
Grade(s): 4th – 10th

Objective: Students will understand that within all 3D solids exist 2D cross sections.
Students will identify 3D solids and 2D shapes.

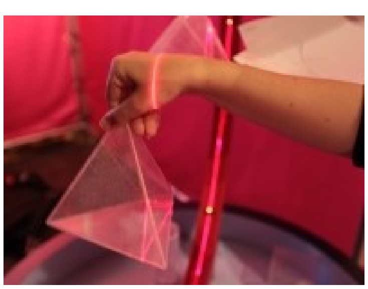
Materials: paper, pencils, 3D solids, chart paper, markers

Procedure:

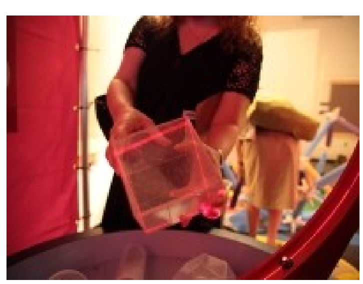
1. Mini-lesson: Demonstrate the exhibit.

Ring of Fire

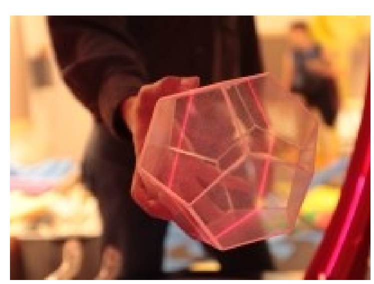
- Before you put a 3D solid through the Ring of Fire, ask students to name the 3D solid. Ask students to name the 2D shapes that they recognize. Ask them what is the difference between a 3D solid and a 2D shape.
- Talk about the difference between the faces of a 3D solid and a cross section of that solid.
- Place 3D solid into the Ring of Fire and show the cross section.
- Talk about the idea of a cross section:
    - cutting the shape into two pieces and looking at inside surface
    - imagine "stamping" the cut shape onto paper to see 2D cross section
    - the cross section is like looking at the white part of the apple when you cut it in two pieces, not the skin
    - you can make only one cut, and it has to go all the way through and cut the shape into two pieces, but the pieces don't have to be equal
- Introduce each of the three dimensional solids at the exhibit (cube, cylinder, cone, tetrahedron, dodecahedron), and ask about the properties of these shapes.
- Ask students what shapes do they think they might find within the 3D solids.
- Record polygons from 3-10 sides as well as circles, ellipses, and parabolas on chart paper.
- For comparison, you can discuss what the shadow of a 3-D object might be. (Note: A shadow may be cast using an overhead projector). How does the shadow compare to the cross section? Under what conditions might they be the same?



Tetrahedron

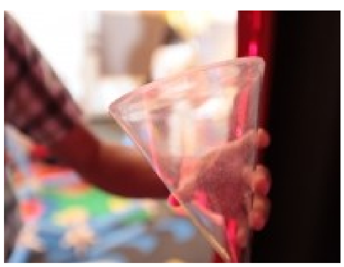


Cube

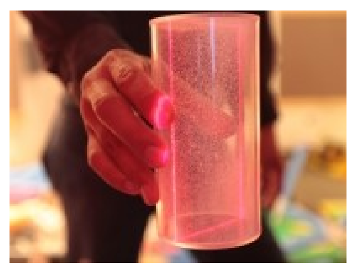


Dodecahedron

2. Practice:
- Break the students into groups of 3-6, depending on group size.
- Hand each group paper and pencil. Tell them to write "Prediction" on one side of the paper and write "Actual" on the other side of the paper.
- Hand out one of the five 3D solids.
- Explain that as a group, they will study the 3D solid and imagine what kind of cross sections they will be able find when they put the solids in the exhibit
- They can draw the shapes on their paper and use the correct names based on the mini-lesson.
- Educators/teachers/chaperones can circulate among groups and give ideas to help them find more cross sections, e.g. What do you think you would see if you cut it this way? Do you think you can get a shape with four sides? How?
- When the group thinks they have found as many shapes as possible, the entire group can use the exhibit to find all of the possible cross sections within the shape.
- An educator should be at the exhibit to help them find all possible shapes.
- When the group has finished with one shape, they trade in their shape for a new one and repeat the process.
- The activity continues until all groups have seen all shapes or until time runs out.
- For comparison if there is additional time, you can ask which 2-D shapes can be the shadow of a tetrahedron, cube, etc., and which 2-D shapes can be both a shadow and a cross section for each of the polyhedra? (Note: A solid object produces a different shadow than a wire or hollow shape.)



Cone



Cylinder

3. Group share:
- Hold up each 3D solid and ask groups what cross sections they found.
- Discuss how a tetrahedron and a cube can have cross sections that have as many sides as the 3D solid has faces. (There is a hexagon in the cube!)
- Discuss how a dodecahedron has cross sections from 3 sides to 10 sides, but not 12 sides because it's impossible to get all 12 sides in the laser plane at once.
- Ask students for any cross sections that surprised them or any other interesting things they noticed.

Notes on this activity: This worked really well with groups. The most important things were to keep adults circulating among the group to keep them on task, and also at the exhibit to manage crowd control. Depending on the size of the groups, only one or at most two groups can be at the exhibit at one time, so the groups that are waiting can sometimes start thinking about a new shape. Students have a hard time distinguishing between cross sections and the shapes on the faces of the solid, so it's important to circulate and get them to predict some of the easier ones, e.g. finding the rectangle in the cylinder, finding a triangle or a rectangle in the cube, etc. Many students will say that they will find a rhombus or a parallelogram. This opens up the discussion about how a square is a parallelogram and a rhombus, and that squares, rhombuses, and rectangles are parallelograms.

Tetrahedron (triangular pyramid) cross sections: triangle, rectangle (parallelogram), square (rhombus, parallelogram), trapezoid
Cube cross sections: triangle, square (rhombus, parallelogram), rectangle (parallelogram), trapezoid, pentagon, hexagon. (The cube has square, rectangle, and hexagon shadows, but not triangles.)
Cylinder cross sections: circle, rectangle (parallelogram), ellipses
Dodecahedron cross sections: triangle, rectangle (parallelogram), trapezoid, pentagon, hexagon, heptagon, octagon, nonagon, decagon
Cone cross sections: circle, ellipse, triangle. There are also hyperbolas and parabolas in the cone, but understanding these and finding the one critical angle which produces a parabola are topics for a separate lesson, which might begin by explaining the difference between this single cone and the mathematician's double cone.

To find the square in the tetrahedron, manipulate it so the slicing plane goes through four edge midpoints. To find a regular hexagon in the cube, manipulate it so the slicing plane goes through six edge midpoints.

Other polyhedra that students can consider are the octahedron (8 faces) and the icosahedron (20 faces).

Title: Polyhedral Puzzle Races
Authors: Math Midway
Topic: Geometry
Grade(s): 2nd and up

Objective: Students will understand that 3D figures can be arranged to create larger 3D figures.

Materials: timer, 27 individual cubes that fit together

Procedure:

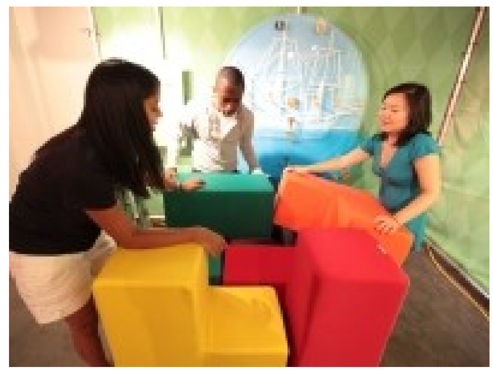
1. Mini-lesson:
   - Show students a cube and ask what smaller 3D shapes can be put together to make a cube.
   - Lead them towards the idea that 27 cubes can be put together to make a larger cube. To help them understand that 27 is 3 cubed, you can ask: Can make a cube with only 20 smaller cubes? What other numbers of cubes can be used to construct a larger solid cube?
   - Show them the Soma cubes and explain the challenge is to fit all 7 pieces together to make a 3x3x3 cube with no holes and nothing sticking out. (Note: there are 240 different solutions, even counting rotations and reflections just once each.)
   - Show students a completed tetraxis and ask them to think about the design. Ask if there seems to be a pattern of repeated pieces. What piece would you start with if you had to rebuild it?
   - Explain that it will be taken apart and the challenge is to put the pieces back together.

Soma cube and tetraxis puzzles

2. Activity:

- Break the students into groups.
- If there are two of each puzzle, two groups are assigned the Soma cube and two groups are assigned the tetraxis puzzles.
- Explain that the two Soma cube groups will race against each other to complete the cube. If there is only one copy of a puzzle, one group will wait as the other group is timed.
- Soma Cube race:
    - If they're having trouble, some good clues include: get the shapes that extend in all three dimensions onto the bottom layer, get 2D pieces on top, save 3-cube piece for last.

Soma cube taken apart



Soma cube put together

- Explain that the two Tetraxis groups will race against each other to see who can complete the Tetraxis.
- Tetraxis race:
    - Students study completed shape.
    - Once they're ready, they take it apart and lay out all the pieces.
    - If they're having trouble, some good clues include: black dots always cover white dots, pieces are always joined at angles as opposed to side by side, white surfaces always face inside, need to start with a base that can have either 3 or 4 pieces.

Tetraxis taken apart

Tetraxis near completion

- The first group to finish the Soma cube can try to build another shape, e.g., the couch, tunnel, or dog, based on the models provided.
- The first group to finish the tetraxis can try again and be timed to see how fast they can do it.
- When all four groups have finished, swap the groups so everyone can do each race.

3. Group share:
   - Ask the students to share which strategies work the best to help them build the 3D shapes.

Notes on this activity: Some groups will get frustrated and need encouragement, but all of them should be able to solve the puzzles.

Title: Organ Function Grinder
Authors: Math Midway
Topic: Algebra
Grade(s): 4th – 9th

Objective: Students will understand that a function is a relationship between values where each input value determines exactly one output value.
Students will conclude that there are multiple ways of arriving at a value.
Students will examine inverse relationships between values.

Materials: index cards (or small pieces of paper), pencils, chart paper, marker

Procedure:

1. Mini-lesson:
   - Demonstrate the exhibit and tell the group that this is the exhibit that will be used for an activity.
   - Introduce/review the idea of the function and go through some examples of functions. Point out that a function only gives one value.
   - Introduce/review the idea of composing functions and go through some examples. Select three students and have each one think of an operation to perform. Line them up and give the first student an index card with a number. Student will perform the operation on the number and write the answer on a new index card. Repeat until the last student performs the operation.
   - Explain that this is what the Organ Function Grinder does.
   - Write on chart paper the eight functions available on the Organ Function Grinder: double, halve, add three, subtract three, square, extract square root, invert, do nothing.
   - Write on chart paper the list of 24 available input numbers. Explain there is no particular mathematical basis for this set. It was just a design choice.
   - Show the group some examples from the chart below.
   - Challenge students to solve additional examples from the chart below. Give them the goal number and the input. Have them find the three functions (from the list of eight possibilities). Note that there may be additional solutions than the ones shown. They can demonstrate the functions with index cards, as above.
   - Have students choose other goal numbers, perhaps finding numbers from around the room, and ask them to generate those numbers by choosing the input number (from the list of 24 possibilities) and the three functions (from the list of eight).

Organ Function Grinder

2. Practice:

- Break the group into teams of four. Each group has index cards and pencils. Each team gives themselves a name.
- Choose a list of twenty consecutive output goals. For younger students this might be the range 20-39. For older students, 50-69 or some other range.
- Tell the team to find a number in the range that they think they can generate. The "captain" of the team selects an input value, while the three remaining members select a function and organize themselves in order so they reach the output goal.

Selecting the input value ticket

Setting the knobs

Cranking the Organ Function Grinder

- Each team member will write down his or her function on an index card.
- The captain will pass the input value written on the index card to the team. Once the function is performed it will be passed to the next member, until the output goal is reached.
- If goal output is reached, the team captain can test it out on the Organ Function Grinder. If it works, then the team records it on a sheet of paper.
- Team members rotate and repeat the process with a different output goal, until as many output goals as possible are reached.
- The activity ends when a team completes all goal outputs or after a set time limit.

Reading the output value ticket

3. Group share:

- Review some of the successful goal outputs.
- Discuss the strategies used to find output numbers.
- Discuss whether any of the goal outputs were possible through multiple functions, or whether some were impossible with the given inputs and functions.
- If the functions are kept the same but their order is changed will the output stay the same? Under what conditions might this occur?
- If all three functions are the "Leave alone" function, what can you say about the input and the output?
- Write in algebraic form several calculations, e.g., square/double/add 3 vs. add 3/double/square

Notes on this activity: It requires some facilitation to make sure only one student from the group is using the exhibit at a time, and the rest of the students can be finding another possible input and function combination. Keeping the students on task is key. Not every number is possible to generate. There are 24 possible inputs and 8^3 possible switch settings, so there are $12{,}288 = 24 \xi 8^3$ possible computations. And many of these give duplicate answers, so there are far fewer distinct outputs possible.

Possible goal outputs and solutions. Select input values and number of goal outputs to suit the needs of the students.

Goal Output	Input	Knob 1	Knob 2	Knob 3
100	97	Add 3	Leave alone	Leave alone
100	10	Leave alone	Leave alone	Square
23	10	Double	Add 3	Leave alone
103	10	Square	Add 3	Leave alone
203	10	Square	Double	Add 3
403	10	Double	Square	Add 3
0.01	10	Square	Invert	Leave alone
100,000,000	10	Square	Square	Square
13	64	Halve	Halve	Subtract 3
14	64	Square root	Add 3	Add 3
1	64	Square root	Halve	Subtract 3
23	13	Subtract 3	Double	Add 3
103	13	Subtract 3	Square	Add 3
49	13	Double	Double	Subtract 3
83	13	Square	Subtract 3	Halve
10,000	13	Subtract 3	Square	Square
6.5	2	Invert	Add 3	Add 3
-5.5	2	Invert	Subtract 3	Subtract 3
-8	2	Subtract 3	Subtract 3	Double
28	2	Add 3	Square	Add 3
256	2	Square	Square	Square

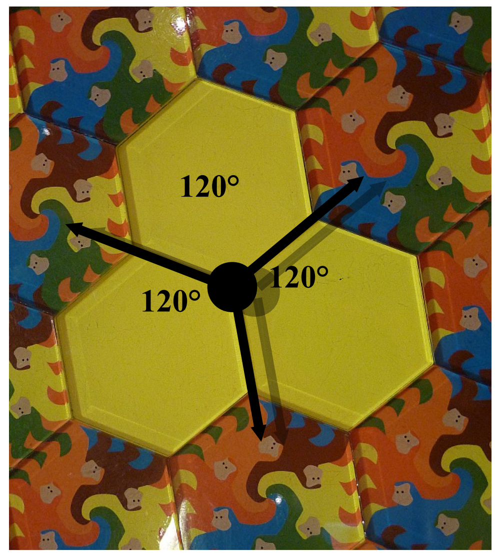
Title: Miles of Tiles 360 Degrees
Authors: Math Midway
Topic: Geometry
Grade(s): 6

Objective: Students will know that polygons have at least 3 or more angles.
Students will understand that there are many different combinations of polygon angles that can surround a point, and that the sum of the angles around a point will always be 360 degrees.

Materials: notebooks, pencils

Procedure:

1. Mini-lesson:



- Review the definition of an angle. An angle is where two rays share an endpoint.
- Draw a circle and show how four 90° angles fit into the circle and how it adds up to 360°. Explain that you can surround a point completely by using angles that add up to 360°. Ask students to come up with another combination of angles that add up to 360°.
- Go through each of the tile shapes and discuss angle measurements around a point using each:
    - triangles: six 60° angles can surround a point
    - squares: four 90° angles
    - hexagons: three 120° angles
    - red rhombus: the acute angle is 36° because ten can surround a point. So the obtuse angle is 180-36 = 144°
    - blue rhombus: the acute angle is 72° degrees because five can surround a point. So the obtuse angle is 180-72 = 108°
- Talk about how we can now surround a point with tiles by making sure everything adds up to 360°:
    - use division: 360/3=120 means that three hexagons will surround a point. Put example on wall.
    - use subtraction 360-90-90-90-90=0 means that four squares will surround a point. Put example on wall
    - use addition/multiplication 60+60+60+60+60+60 or 60x6=360 means that six triangles will surround a point. Put example on wall
    - We can surround a point with different kind of shapes as well, 360-60-90-120-90=0. So one triangle, two squares, and a hexagon will surround a point. Demonstrate on wall.

2. Practice:
   - Break group into teams of 3-4 students.
   - Each team will receive notebooks and pencils.
   - Explain that they can try different strategies (subtraction, division, etc) try to find new combinations of the available angles that will surround a point completely.
   - Once a group has a new combination, send one student to make that combination up at the wall.
   - Challenge groups to try to surround a point using 3 polygons, 4 polygons, etc. up to 10 polygons.
   - Continue to find new combinations as time permits.

3. Group share:
   - Go through the different examples that the students came up with and verify that all the angles add up exactly to 360°, show that some might use same shapes but in a different order.
   - Talk about combinations they tried to make that wouldn't work and why.
   - Talk about combinations that can be repeated at every point (tessellations) and that not every combination that surrounds a point can be extended to fill the plane. This might lead into a discussion about the exterior angles of polygons and that their sum is also 360° if one exterior angle is chosen from each vertex.

Notes on this activity: This was designed to concentrate on arithmetic, so we focused on the angle measurements. Avoid handing out the tiles to groups before they have come up with angle measurements since they will just rearrange the tiles and not think about the angle measurements.

Examples of 3 to 10 polygons that can surround a point. When rhombi are used, this assumes you choose the acute or obtuse angles in the rhombi appropriately to make the sum of the angles be 360.

3 polygons: [3 hexagons], [2 red rhombuses (obtuse angles), 1 blue rhombus (acute angle)], [2 blue rhombuses (obtuse angles), 1 red rhombus (obtuse angle)]
4 polygons: [4 squares], [2 squares, 2 blue rhombuses], [2 squares, 2 red rhombuses], [1 hexagon, 1 triangle, 2 squares]
5 polygons: [5 blue rhombuses], [2 squares, 2 blue rhombuses, 1 red rhombus], [1 hexagon, 4 triangles], [3 triangles, 2 squares]
6 polygons: [6 triangles], [2 red rhombuses, 4 blue rhombuses]
7 polygons: [5 red rhombuses, 2 squares], [7 red rhombuses], [5 red rhombuses, 1 triangle, 1 hexagon]
8 polygons: [5 red rhombuses, 3 triangles], [7 red rhombuses, 1 blue rhombus]
9 polygons: [8 red rhombuses, 1 blue rhombus]
10 polygons: [10 red rhombuses]

Title: Miles of Tiles Patterns
Authors: Math Midway
Topic: Geometry
Grade(s): 2nd – 3rd

Objective: Students will understand that a pattern is a structure that is built by repetition.

Materials:

Procedure:

1. Mini-lesson:
    - Introduce/review the idea of a pattern as being based on a rule that you could repeat. Show examples of repeating polygons, or brings students up in front of the room to physically represent a pattern (e.g., facing forward, facing backward).
    - Explain that a pattern does not have to be straight across. Show examples of a tessellation pattern, using just squares and triangles.
    - Ask students to identify the polygons that make up the patterns: triangle, square, hexagon, rhombus.
    - Tell them that their goal is to take the patterns on the tile wall and extend them to make tessellations.

Tile Wall

2. Practice:
    - Explain that once they are assigned to one side of the tile wall, they can pick a pattern to copy and extend.
    - Send students to each side of the tile wall to look at the patterns at the top.
    - When they are finished extending one pattern they can select another one on their side of the wall and repeat the process.
    - Educators/teachers/chaperones can walk around the wall and talk to the students about what the rule is for their pattern and how they can keep their pattern going.

Double-sided tile wall

3. Group share:
   - Look at the patterns that the students made and compare them with the original smaller versions.
   - Ask students to share the strategies that they used to make their patterns bigger.

Notes on this activity: There are many ways to start and extend the patterns. One example was a tessellation of rhombuses and squares where the student extended the pattern by magnifying it—each rhombus was replaced by 4 rhombi and each square was replaced by 4 squares, so the student made the pattern jumbo.

Title: The Greedy Triangle
Authors: Math Midway
Topic: Geometry
Grade(s): K – 1st

Objective: Students will understand that shapes can be found all around us.
Students will identify the following polygons: triangle, quadrilaterals, pentagons, and hexagons.

Materials: Chart paper, markers, paper, pencils, book: *The Greedy Triangle*

Procedure:

1. Mini-lesson:
    - Introduce the book, *The Greedy Triangle* to the students.
    - Tell them that there will be many shapes in the book, and ask them what shapes could they possibly see.
    - Record on chart paper.
    - Tell them that each time they see one of the shapes on the chart paper, quietly raise their hands and tell us which one they saw.
    - Ask them to explain how they knew it was that shape.
    - Tell students that they need to practice what they did with the book, with what they see in the room. Look for the shapes in the room.

2. Practice:
    - Break group into smaller groups and assign one chaperone to each group.
    - Each group will visit one exhibit and look for examples of the different shapes from the book: triangles, quadrilaterals, pentagons, and hexagons.
    - Chaperones can record the shapes and location on sheets of paper.

Polygons at the Wheel of Chance

Polygons at Pedal on the Petal

Polygons at Ring of Fire

3. Group share:
    - Ask students to share some of the shapes that they found in the room.
    - Review what happens in *The Greedy Triangle*. He went to a shapeshifter to get more sides and more angles. There's something like the shapeshifter in the Math Midway, and it's the Ring of Fire exhibit.
    - Educator does a demonstration with the tetrahedron, cube, and dodecahedron showing the cross sections all the way from 3 sides to 10 sides, letting the

children count the sides with each new shape.

- Encourage the group to try this exhibit during their free time in the exhibition to see how many of the shapes they can make, and encourage them to keep looking out for the different shapes in the museum and out in the world.

Notes on this activity: Because the students have so many distractions in the room, it is important for chaperones to keep them on task.